O LEVEL 4037

ADDITIONAL MATHEMATICS

TOPICAL PAPER 2

WITH MARK SCHEME

June 2012 – November 2020

FOR CAMBRIDGE 2022 and onwards EXAMS

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Content Page

Assessment overview

All candidates take **two** components.

Candidates are eligible for grades A* to E.

All candidates take:		**and:**	
Paper 1	2 hours 50%	**Paper 2**	2 hours 50%
80 marks Candidates answer all questions Scientific calculators are required Externally assessed		80 marks Candidates answer all questions Scientific calculators are required Externally assessed	

Information on availability is in the **Before you start** section.

List of formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

3 Subject content

This syllabus gives you the flexibility to design a course that will interest, challenge and engage your learners. Where appropriate you are responsible for selecting resources and examples to support your learners' study. These should be appropriate for the learners' age, cultural background and learning context as well as complying with your school policies and local legal requirements.

Knowledge of the content of Cambridge O Level Mathematics (or an equivalent syllabus) is assumed.

Cambridge O Level material which is not included in the subject content will not be tested directly but it may be required in response to questions on other topics.

Proofs of results will not be required unless specifically mentioned in the syllabus.

Candidates will be expected to be familiar with the scientific notation for the expression of compound units, e.g. $5\,\text{ms}^{-1}$ for 5 metres per second.

1 Functions

- understand the terms: function, domain, range (image set), one-one function, inverse function and composition of functions
- use the notation $\text{f}(x) = \sin x$, $\text{f}: x \mapsto \lg x$, $(x > 0)$, $\text{f}^{-1}(x)$ and $\text{f}^2(x)$ $[= \text{f}(\text{f}(x))]$
- understand the relationship between $y = \text{f}(x)$ and $y = |\text{f}(x)|$, where $\text{f}(x)$ may be linear, quadratic or trigonometric
- explain in words why a given function is a function or why it does not have an inverse
- find the inverse of a one-one function and form composite functions
- use sketch graphs to show the relationship between a function and its inverse

2 Quadratic functions

- find the maximum or minimum value of the quadratic function $\text{f} : x \mapsto ax^2 + bx + c$ by any method
- use the maximum or minimum value of $\text{f}(x)$ to sketch the graph or determine the range for a given domain
- know the conditions for $\text{f}(x) = 0$ to have:
  (i) two real roots, (ii) two equal roots, (iii) no real roots
  and the related conditions for a given line to
  (i) intersect a given curve, (ii) be a tangent to a given curve, (iii) not intersect a given curve
- solve quadratic equations for real roots and find the solution set for quadratic inequalities

3 Equations, inequalities and graphs

- solve graphically or algebraically equations of the type $|ax + b| = c$ $(c \geqslant 0)$ and $|ax + b| = |cx + d|$
- solve graphically or algebraically inequalities of the type
  $|ax + b| > c$ $(c \geqslant 0)$, $|ax + b| \leqslant c$ $(c > 0)$ and $|ax + b| \leqslant |cx + d|$
- use substitution to form and solve a quadratic equation in order to solve a related equation
- sketch the graphs of cubic polynomials and their moduli, when given in factorised form
  $y = k(x - a)(x - b)(x - c)$
- solve cubic inequalities in the form $k(x - a)(x - b)(x - c) \leqslant d$ graphically

4 Indices and surds

- perform simple operations with indices and with surds, including rationalising the denominator

5 Factors of polynomials

- know and use the remainder and factor theorems
- find factors of polynomials
- solve cubic equations

6 Simultaneous equations

- solve simple simultaneous equations in two unknowns by elimination or substitution

7 Logarithmic and exponential functions

- know simple properties and graphs of the logarithmic and exponential functions including $\ln x$ and e^x (series expansions are not required) and graphs of $ke^{nx} + a$ and $k\ln(ax + b)$ where n, k, a and b are integers
- know and use the laws of logarithms (including change of base of logarithms)
- solve equations of the form $a^x = b$

8 Straight line graphs

- interpret the equation of a straight line graph in the form $y = mx + c$
- transform given relationships, including $y = ax^n$ and $y = Ab^x$, to straight line form and hence determine unknown constants by calculating the gradient or intercept of the transformed graph
- solve questions involving mid-point and length of a line
- know and use the condition for two lines to be parallel or perpendicular, including finding the equation of perpendicular bisectors

9 Circular measure

- solve problems involving the arc length and sector area of a circle, including knowledge and use of radian measure

10 Trigonometry

- know the six trigonometric functions of angles of any magnitude (sine, cosine, tangent, secant, cosecant, cotangent)
- understand amplitude and periodicity and the relationship between graphs of related trigonometric functions, e.g. $\sin x$ and $\sin 2x$
- draw and use the graphs of
  $y = a\sin bx + c$
  $y = a\cos bx + c$
  $y = a\tan bx + c$
  where a is a positive integer, b is a simple fraction or integer (fractions will have a denominator of 2, 3, 4, 6 or 8 only), and c is an integer
- use the relationships
  $\sin^2 A + \cos^2 A = 1$
  $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$
  $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$
- solve simple trigonometric equations involving the six trigonometric functions and the above relationships (not including general solution of trigonometric equations)
- prove simple trigonometric identities

11 Permutations and combinations

- recognise and distinguish between a permutation case and a combination case
- know and use the notation $n!$ (with $0! = 1$), and the expressions for permutations and combinations of n items taken r at a time
- answer simple problems on arrangement and selection (cases with repetition of objects, or with objects arranged in a circle, or involving both permutations and combinations, are excluded)

12 Series

- use the Binomial Theorem for expansion of $(a + b)^n$ for positive integer n
- use the general term $\binom{n}{r} a^{n-r} b^r$, $0 \leqslant r \leqslant n$ (knowledge of the greatest term and properties of the coefficients is not required)
- recognise arithmetic and geometric progressions
- use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions
- use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression

13 Vectors in two dimensions

- use vectors in any form, e.g. $\begin{pmatrix} a \\ b \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{p}$, $a\mathbf{i} - b\mathbf{j}$
- know and use position vectors and unit vectors
- find the magnitude of a vector; add and subtract vectors and multiply vectors by scalars
- compose and resolve velocities

14 Differentiation and integration

- understand the idea of a derived function
- use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ $\left[= \frac{d}{dx}\left(\frac{dy}{dx}\right)\right]$
- use the derivatives of the standard functions x^n (for any rational n), $\sin x$, $\cos x$, $\tan x$, e^x, $\ln x$, together with constant multiples, sums and composite functions of these
- differentiate products and quotients of functions
- apply differentiation to gradients, tangents and normals, stationary points, connected rates of change, small increments and approximations and practical maxima and minima problems
- use the first and second derivative tests to discriminate between maxima and minima
- understand integration as the reverse process of differentiation
- integrate sums of terms in powers of x including $\frac{1}{x}$ and $\frac{1}{ax+b}$
- integrate functions of the form $(ax + b)^n$ for any rational n, $\sin(ax + b)$, $\cos(ax + b)$, e^{ax+b}
- evaluate definite integrals and apply integration to the evaluation of plane areas
- apply differentiation and integration to kinematics problems that involve displacement, velocity and acceleration of a particle moving in a straight line with variable or constant acceleration, and the use of x–t and v–t graphs

TOPIC 1: QUADRATIC FUNCTIONS

1 **4037/22/M/J/12/Q3**

Find the values of m for which the line $y = mx - 5$ is a tangent to the curve $y = x^2 + 3x + 4$.

[5]

2 **4037/23/O/N/12/Q5**

Find the set of values of m for which the line $y = mx + 2$ does not meet the curve $y = mx^2 + 7x + 11$.

[6]

3 4037/23/O/N/13/Q3

Find the set of values of k for which the line $y = 3x - k$ does not meet the curve $y = kx^2 + 11x - 6$.

[6]

4 **4037/21/M/J/14/Q5**

(i) Express $2x^2 - x + 6$ in the form $p(x-q)^2 + r$, where p, q and r are constants to be found.

[3]

(ii) Hence state the least value of $2x^2 - x + 6$ and the value of x at which this occurs.

[2]

5 **4037/22/M/J/14/Q2**

Find the values of k for which the line $y + kx - 2 = 0$ is a tangent to the curve $y = 2x^2 - 9x + 4$.

[5]

6 **4037/22/M/J/14/Q4**

(i) Express $12x^2 - 6x + 5$ in the form $p(x-q)^2 + r$, where p, q and r are constants to be found.

[3]

(ii) Hence find the greatest value of $\dfrac{1}{12x^2 - 6x + 5}$ and state the value of x at which this occurs.

[2]

7 4037/23/O/N/15/Q2

Find the values of k for which the line $y = 2x + k + 2$ cuts the curve $y = 2x^2 + (k + 2)x + 8$ in two distinct points.

[6]

8 **4037/22/O/J/16/Q9**

The line $y = kx - 4$, where k is a positive constant, passes through the point P (0, −4) and is a tangent to the curve $x^2 + y^2 - 2y = 8$ at the point T. Find

(i) the value of k,

[5]

(ii) the coordinates of T,

[3]

(iii) the length of TP.

[2]

9 4037/22/O/N/18/Q10

Two lines are tangents to the curve $y = 12 - 4x - x^2$. The equation of each tangent is of the form $y = 2k + 1 - kx$, where k is a constant.

(i) Find the two possible values of k.

[5]

(ii) Find the coordinates of the point of intersection of the two tangents.

[4]

10 4037/23/O/N/18/Q3

(i) Write $8+7x-x^2$ in the form $a-(x-b)^2$, where a and b are constants.

[3]

(ii) Hence state the maximum value of $8+7x-x^2$ and the value of x at which it occurs.

[2]

(iii) Using your answer to **part (i)**, or otherwise, solve the equation $8+7z^2-z^4=0$.

[3]

11 4037/23/O/N/18/Q11

A line with equation $y = -5x + k + 5$ is a tangent to a curve with equation $y = 7 - kx - x^2$.

(i) Find the two possible values of k.

[5]

(ii) Find, for **each** of your values of k,

- the equation of the tangent
- the equation of the curve
- the coordinates of the point of contact of the tangent and the curve.

[5]

(iii) Find the distance between the two points of contact.

[2]

12 **4037/22/M/J/19/Q5**

(i) Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants.

[3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs.

[2]

13 4037/22/O/N/19/Q4

Find the values of k for which the line $y = kx + 3$ does not meet the curve $y = x^2 + 5x + 12$.

[5]

14 4037/23/O/N/19/Q4

(i) Given that $y = 2x^2 - 4x - 7$, write y in the form $a(x-b)^2 + c$, where a, b and c are constants.

[3]

(ii) Hence write down the minimum value of y and the value of x at which it occurs.

[2]

(iii) Using your answer to **part (i)**, solve the equation $2p - 4\sqrt{p} - 7 = 0$, giving your answer correct to 2 decimal places.

[3]