O LEVEL 4037

ADDITIONAL MATHEMATICS

TOPICAL PAPER 1

WITH MARK SCHEME

June 2011 – November 2020

FOR CAMBRIDGE 2022 and onwards EXAMS

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Assessment overview

All candidates take **two** components.

Candidates are eligible for grades A* to E.

All candidates take:		**and:**	
Paper 1	2 hours 50%	**Paper 2**	2 hours 50%
80 marks Candidates answer all questions Scientific calculators are required Externally assessed		80 marks Candidates answer all questions Scientific calculators are required Externally assessed	

Information on availability is in the **Before you start** section.

List of formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

3 Subject content

This syllabus gives you the flexibility to design a course that will interest, challenge and engage your learners. Where appropriate you are responsible for selecting resources and examples to support your learners' study. These should be appropriate for the learners' age, cultural background and learning context as well as complying with your school policies and local legal requirements.

Knowledge of the content of Cambridge O Level Mathematics (or an equivalent syllabus) is assumed.

Cambridge O Level material which is not included in the subject content will not be tested directly but it may be required in response to questions on other topics.

Proofs of results will not be required unless specifically mentioned in the syllabus.

Candidates will be expected to be familiar with the scientific notation for the expression of compound units, e.g. $5\,\text{ms}^{-1}$ for 5 metres per second.

1 Functions

- understand the terms: function, domain, range (image set), one-one function, inverse function and composition of functions
- use the notation $\text{f}(x) = \sin x$, $\text{f}: x \mapsto \lg x$, $(x > 0)$, $\text{f}^{-1}(x)$ and $\text{f}^2(x)$ $[= \text{f}(\text{f}(x))]$
- understand the relationship between $y = \text{f}(x)$ and $y = |\text{f}(x)|$, where $\text{f}(x)$ may be linear, quadratic or trigonometric
- explain in words why a given function is a function or why it does not have an inverse
- find the inverse of a one-one function and form composite functions
- use sketch graphs to show the relationship between a function and its inverse

2 Quadratic functions

- find the maximum or minimum value of the quadratic function $\text{f} : x \mapsto ax^2 + bx + c$ by any method
- use the maximum or minimum value of $\text{f}(x)$ to sketch the graph or determine the range for a given domain
- know the conditions for $\text{f}(x) = 0$ to have:
  (i) two real roots, (ii) two equal roots, (iii) no real roots
  and the related conditions for a given line to
  (i) intersect a given curve, (ii) be a tangent to a given curve, (iii) not intersect a given curve
- solve quadratic equations for real roots and find the solution set for quadratic inequalities

3 Equations, inequalities and graphs

- solve graphically or algebraically equations of the type $|ax + b| = c$ $(c \geqslant 0)$ and $|ax + b| = |cx + d|$
- solve graphically or algebraically inequalities of the type $|ax + b| > c$ $(c \geqslant 0)$, $|ax + b| \leqslant c$ $(c > 0)$ and $|ax + b| \leqslant |cx + d|$
- use substitution to form and solve a quadratic equation in order to solve a related equation
- sketch the graphs of cubic polynomials and their moduli, when given in factorised form $y = k(x - a)(x - b)(x - c)$
- solve cubic inequalities in the form $k(x - a)(x - b)(x - c) \leqslant d$ graphically

4 Indices and surds

- perform simple operations with indices and with surds, including rationalising the denominator

5 Factors of polynomials

- know and use the remainder and factor theorems
- find factors of polynomials
- solve cubic equations

6 Simultaneous equations

- solve simple simultaneous equations in two unknowns by elimination or substitution

7 Logarithmic and exponential functions

- know simple properties and graphs of the logarithmic and exponential functions including $\ln x$ and e^x (series expansions are not required) and graphs of $ke^{nx} + a$ and $k\ln(ax + b)$ where n, k, a and b are integers
- know and use the laws of logarithms (including change of base of logarithms)
- solve equations of the form $a^x = b$

8 Straight line graphs

- interpret the equation of a straight line graph in the form $y = mx + c$
- transform given relationships, including $y = ax^n$ and $y = Ab^x$, to straight line form and hence determine unknown constants by calculating the gradient or intercept of the transformed graph
- solve questions involving mid-point and length of a line
- know and use the condition for two lines to be parallel or perpendicular, including finding the equation of perpendicular bisectors

9 Circular measure

- solve problems involving the arc length and sector area of a circle, including knowledge and use of radian measure

10 Trigonometry

- know the six trigonometric functions of angles of any magnitude (sine, cosine, tangent, secant, cosecant, cotangent)
- understand amplitude and periodicity and the relationship between graphs of related trigonometric functions, e.g. $\sin x$ and $\sin 2x$
- draw and use the graphs of
  $y = a\sin bx + c$
  $y = a\cos bx + c$
  $y = a\tan bx + c$
  where a is a positive integer, b is a simple fraction or integer (fractions will have a denominator of 2, 3, 4, 6 or 8 only), and c is an integer
- use the relationships
  $\sin^2 A + \cos^2 A = 1$
  $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$
  $\dfrac{\sin A}{\cos A} = \tan A$, $\dfrac{\cos A}{\sin A} = \cot A$
- solve simple trigonometric equations involving the six trigonometric functions and the above relationships (not including general solution of trigonometric equations)
- prove simple trigonometric identities

11 Permutations and combinations

- recognise and distinguish between a permutation case and a combination case
- know and use the notation $n!$ (with $0! = 1$), and the expressions for permutations and combinations of n items taken r at a time
- answer simple problems on arrangement and selection (cases with repetition of objects, or with objects arranged in a circle, or involving both permutations and combinations, are excluded)

12 Series

- use the Binomial Theorem for expansion of $(a + b)^n$ for positive integer n
- use the general term $\binom{n}{r}a^{n-r}b^r$, $0 \leqslant r \leqslant n$ (knowledge of the greatest term and properties of the coefficients is not required)
- recognise arithmetic and geometric progressions
- use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions
- use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression

13 Vectors in two dimensions

- use vectors in any form, e.g. $\begin{pmatrix} a \\ b \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{p}$, $a\mathbf{i} - b\mathbf{j}$
- know and use position vectors and unit vectors
- find the magnitude of a vector; add and subtract vectors and multiply vectors by scalars
- compose and resolve velocities

14 Differentiation and integration

- understand the idea of a derived function
- use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ $\left[= \frac{d}{dx}\left(\frac{dy}{dx}\right)\right]$
- use the derivatives of the standard functions x^n (for any rational n), $\sin x$, $\cos x$, $\tan x$, e^x, $\ln x$, together with constant multiples, sums and composite functions of these
- differentiate products and quotients of functions
- apply differentiation to gradients, tangents and normals, stationary points, connected rates of change, small increments and approximations and practical maxima and minima problems
- use the first and second derivative tests to discriminate between maxima and minima
- understand integration as the reverse process of differentiation
- integrate sums of terms in powers of x including $\frac{1}{x}$ and $\frac{1}{ax+b}$
- integrate functions of the form $(ax + b)^n$ for any rational n, $\sin(ax + b)$, $\cos(ax + b)$, $e^{ax + b}$
- evaluate definite integrals and apply integration to the evaluation of plane areas
- apply differentiation and integration to kinematics problems that involve displacement, velocity and acceleration of a particle moving in a straight line with variable or constant acceleration, and the use of x–t and v–t graphs

TOPIC 1: QUADRATIC FUNCTIONS

1 4037/11/M/J/12/Q3

Find the set of values of k for which the line $y = 2x + k$ cuts the curve $y = x^2 + kx + 5$ at two distinct points.

[6]

2 **4037/12/M/J/13/Q4**

Find the set of values of k for which the curve $y = 2x^2 + kx + 2k - 6$ lies above the x-axis for all values of x.

[4]

3 **4037/12/O/N/13/Q2**

Find the set of values of k for which the curve $y = (k + 1)x^2 - 3x + (k + 1)$ lies below the x-axis.

[4]

4 **4037/11/M/J/14/Q4**

Find the set of values of k for which the line $y = k(4x - 3)$ does not intersect the curve $y = 4x^2 + 8x - 8$.

[5]

5 4037/12/M/J/15/Q1

Given that the graph of $y = (2k + 5)x^2 + kx + 1$ does not meet the x-axis, find the possible values of k.

[4]

6 **4037/12/O/N/15/Q1**

Find the range of values of k for which the equation $kx^2 + k = 8x - 2xk$ has 2 real distinct roots.

[4]

7 **4037/13/O/N/16/Q3**

(i) Given that $3x^2 + p(1-2x) = -3$, show that, for x to be real, $p^2 - 3p - 9 \geqslant 0$.

[3]

(ii) Hence find the set of values of p for which x is real, expressing your answer in exact form.

[3]

8 **4037/11/M/J/17/Q1**

The line $y = kx - 5$, where k is a positive constant, is a tangent to the curve $y = x^2 + 4x$ at the point A.

(i) Find the exact value of k.

[3]

(ii) Find the gradient of the normal to the curve at the point A, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants.

[2]

9 **4037/13/O/N/17/Q3**

Find the set of values of k for which the equation $kx^2 + 3x - 4 + k = 0$ has no real roots.

[4]

10 **4037/12/M/J/18/Q2**

Find the values of k for which the line $y = 1 - 2kx$ does not meet the curve $y = 9x^2 - (3k + 1)x + 5$.

[5]

1	$x^2 + x(k - 2) + (5 - k) = 0$ Using '$b^2 > 4ac$', $(k - 2^2) > 4(5 - k)$ $k^2 > 16$ $k > 4$, $k < -4$	**M1** **DM1** **A1** **A1** **A1, A1** **[6]**	**M1** for equating line and curve **DM1** for use of $b^2 > 4ac$ $b = k - 2$ and c = $5 - k$ Accept < = ≥ ≤ etc. **A1** for each

2	$2x^2 + kx + 2k - 6 = 0$ has no real roots $k^2 - 16k + 48 < 0$ $(k - 4)(k - 12) < 0$	**M1** **DM1**	**M1** for attempted use of $b^2 - 4ac$ **DM1** for attempt to obtain critical values from a 3 term quadratic
	Critical values 4 and 12 $4 < k < 12$ or $k > 4$ **and** $k < 12$	**A1** **A1**	**A1** for both critical values **A1** for correct final answer
	OR $\left(x + \frac{k}{4}\right)^2 - \frac{k^2}{16} + k - 3 = 0$	**[M1]**	**M1** for attempting to complete the square and obtain a 3 term quadratic
	$-\frac{k^2}{16} + k - 3 > 0$ so $k^2 - 16k + 48 < 0$		Then as **EITHER**
	OR $\frac{dy}{dx} = 4x + k$	**[M1**	**M1** for differentiation, equating to zero and obtaining a quadratic equation in x
	When $\frac{dy}{dx} = 0$, $k = -4x$ By substitution $x^2 + 4x + 3 < 0$ leading to $x = -1$, $k = 4$	**DM1**	**DM1** for attempt to obtain critical values of k from a 3 term quadratic in x followed by substitution to obtain a value for k
	$x = -3$, $k = 12$ $4 < k < 12$ or $k > 4$ **and** $k < 12$	**A1** **A1]**	**A1** for both critical values **A1** for correct final answer
	OR $\frac{dy}{dx} = 4x + k$	**[M1]**	**M1** for differentiation, equating to zero and obtaining a quadratic equation in k
	When $\frac{dy}{dx} = 0$, $x = -\frac{k}{4}$ leading to $k^2 - 16k + 48 < 0$		Then as **EITHER**

3	Using $b^2 - 4ac$, $9 = 4(k+1)^2$ $4k^2 + 8k - 5 = 0$ $k = -\frac{5}{2}, \left(\frac{1}{2}\right)$ To be below the x-axis $k < -\frac{5}{2}$	M1 DM1 A1 A1 [4]	M1 for any use of $b^2 - 4ac$ DM1 for solution of their quadratic in k A1 for critical value(s), $\frac{1}{2}$ not necessary A1 for $k < -\frac{5}{2}$ only
	Or $\frac{dy}{dx} = 2(k+1)x - 3$ when $\frac{dy}{dx} = 0$, $x = \frac{3}{2(k+1)}$ $\therefore y = (k+1)\frac{9}{4(k+1)^2} - \frac{9}{2(k+1)} + (k+1)$ To lie under the x-axis, $y < 0$ $\therefore (k+1)\frac{9}{4(k+1)^2} - \frac{9}{2(k+1)} + (k+1) < 0$ leading to $9 = 4(k+1)^2$ or equivalent then as for previous method	M1	M1 for a complete method to this point.

4	$k(4x-3) = 4x^2 + 8x - 8$ $4x^2 + x(8-4k) + 3k - 8 = 0$	M1	M1 for equating the line and the curve and attempt to obtain a quadratic equation in k
	$b^2 - 4ac = (8-4k)^2 - 16(3k-8)$ $= 16k^2 - 112k + 192$	DM1	DM1 for use of $b^2 - 4ac$ with k
	$b^2 - 4ac < 0$, $k^2 - 7k + 12 < 0$	DM1	DM1 for solution of a 3 term quadratic equation, dependent on both previous M marks
	critical values $k = 3, 4$	A1	A1 for both critical values
	$\therefore 3 < k < 4$	A1	A1 for the range

5	$k^2 - 4(2k+5) \quad (<0)$ $k^2 - 8k - 20 \quad (<0)$	M1	use of $b^2 - 4ac$, (not as part of quadratic formula unless isolated at a later stage) with correct values for a, b and c Do not need to see < at this point
	$(k-10)(k+2) \quad (<0)$	M1	attempt to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2 < k < 10$	A1	correct range
	Alternative 1:		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(2k+5)x + k$	M1	attempt to differentiate, equate to zero and substitute x value back in to obtain a y value
	When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0,\ x = \frac{-k}{2(2k+5)},\ y = \frac{8k+20-k^2}{4(2k+5)}$	M1	consider $y = 0$ in order to obtain critical values
	When $y = 0$, obtain critical values of 10 and –2	A1	correct critical values
	$-2 < k < 10$	A1	correct range
	Alternative 2:		
	$y = (2k+5)\left(\left(x + \frac{k}{2(2k+5)}\right)^2 - \frac{k^2}{4(2k+5)}\right) + 1$	M1	attempt to complete the square and consider $'1 - \frac{k^2}{4(2k+5)}'$
	Looking at $1 - \frac{k^2}{4(2k+5)} = 0$ leads to	M1	attempt to solve above = to 0, to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2 < k < 10$	A1	correct range

6	$kx^2 + (2k-8)x + k = 0$	M1	for attempt to obtain a 3 term quadratic in the form $ax^2 + bx + c = 0$, where b contains a term in k and a constant
	$b^2 - 4ac > 0$ so $(2k-8)^2 - 4k^2 (>0)$	DM1	for use of $b^2 - 4ac$
	$4k^2 - 32k + 64 - 4k^2 (>0)$	DM1	for attempt to simplify and solve for k
	leading to $k < 2$ only	A1	A1 must have correct sign

7 (i)	$3x^2 - 2xp + (p+3) = 0$ $(-2p)^2 - 4 \times 3 \times (p+3) \geqslant 0$ oe	**M1** **DM1**	for obtaining a 3-term quadratic in the form $ax^2 + bx + c(=0)$ for correct substitution of *their* a, b and c into '$b^2 - 4ac$' and use of discriminant.
	$p^2 \geqslant 3(p+3)$ or $4p^2 - 12p - 36 \geqslant 0$ $p^2 - 3p - 9 \geqslant 0$	**A1**	for full correct working, $\geqslant$ the only sign used, $\geqslant$ used before division by 4 and $\geqslant$ used in answer line and penultimate line.
(ii)	Correct method of solution $p^2 - 3p - 9 = 0$ leading to critical values	**M1**	for correct substitution in the quadratic formula or for correct attempt to complete the square. (allow 1 sign error in either method)
	$p = \frac{3 \pm 3\sqrt{5}}{2}$	**A1**	for both correct critical values
	$p \leqslant \frac{3 - 3\sqrt{5}}{2}$, $p \geqslant \frac{3 + 3\sqrt{5}}{2}$	**A1**	for correct range

8 (i)	$kx - 5 = x^2 + 4x$ $x^2 + (4-k)x + 5 = 0$	**M1**	equating line and curve equation and collecting terms to form an equation of the form $ax^2 + bx + c = 0$ x terms must be gathered together, maybe implied by later work
	For a tangent $(4-k)^2 = 20$	**DM1**	correct use of discriminant
	$k = 4 + 2\sqrt{5}$	**A1**	Accept $k = 4 + \sqrt{20}$
	Alternative Gradient of line = k Gradient of curve = $\frac{dy}{dx} = 2x + 4$ Equating: $k = 2x + 4$	**M1**	
	substitution of $k = 2x + 4$ or $x = \frac{k-4}{2}$ in $kx - 5 = x^2 + 4$ and simplify to a quadratic equation in k or x	**DM1**	
	$k = 4 + 2\sqrt{5}$	**A1**	Accept $k = 4 + \sqrt{20}$

(ii)	Normal gradient = $-\dfrac{1}{4+2\sqrt{5}} \times \dfrac{4-2\sqrt{5}}{4-2\sqrt{5}}$	**M1**	use of negative reciprocal and attempt to rationalise using a form of $a-b\sqrt{5}$ or $a-\sqrt{20}$ or *their* equivalent from (i)
	$= -\dfrac{4-2\sqrt{5}}{-4}$ oe $= 1-\dfrac{\sqrt{5}}{2}$	**A1**	$-\dfrac{4-2\sqrt{5}}{-4}$ oe leading to $1-\dfrac{\sqrt{5}}{2}$

9	$9 < 4k(k-4)$ $4k^2-16k-9$	**M1**	use of the discriminant with correct values
	$(2k-9)(2k+1)$	**M1**	**M1dep** for solution of *their* quadratic to obtain critical values
	Critical values $\dfrac{9}{2}$, $-\dfrac{1}{2}$	**A1**	
	$k < -\dfrac{1}{2}$, $k > \dfrac{9}{2}$	**A1**	

10	For an attempt to obtain an equation in *x* only	**M1**	
	$9x^2-(k+1)x+4=0$	**A1**	correct 3 term equation
	$(k+1)^2-(4\times 9\times 4)$	**M1**	**M1dep** for correct use of b^2-4ac oe
	Critical values $k=11$, $k=-13$	**A1**	
	$-13 < k < 11$	**A1**	For the correct range

TOPIC 2: SIMULTANEOUS EQUATIONS

1 **4037/12/M/J/12/Q4**

Solve the simultaneous equations $5x + 3y = 2$ and $\frac{2}{x} - \frac{3}{y} = 1$.

[5]

2 4037/12/M/J/13/Q5

The line $3x + 4y = 15$ cuts the curve $2xy = 9$ at the points A and B. Find the length of the line AB.

[6]

3 **4037/13/O/N/14/Q2**

The line $4y = x + 8$ cuts the curve $xy = 4 + 2x$ at the points A and B. Find the exact length of AB.

[5]