Cambridge A Level 9709

MATHEMATICS

PAPER 4 (M1)
MECHANICS 1 TOPICAL

WITH ANSWERS

JUNE 2011 – JUNE 2023

FOR CAMBRIDGE 2022-2024 and onwards EXAMS

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Book Title: A Level Pure Mathematics Topical Paper 4 (M1)

Book Code 1101

Edition: 3rd Edition | 2nd Impression

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Years Range: June 2011 – June 2023

Syllabus: Latest 2023-25 Syllabus

Published by: STUDENTS RESOURCE **Airport Road** 0423-5700707

Price: 1600/-

Preface

This Mechanics M1 Question Bank is the best preparation tool for practice of Cambridge 2022 and onwards exams.

This new series covers the entire new syllabus for CAIE Mathematics (9709/M1). All the Questions are extracted from the past papers of Cambridge Assessment International Examinations A-Level from 2002 to 2020, including all variants as I tried to provide maximum number of questions in each topic for their detailed practice.

I am also thankful to all my friends and colleagues who encouraged me, especially **Sir Jawad Saeed** who gave me some valuable suggestions for developing this book.

I have added formulae sheets and tables at the start of every topic as helping tool for students., if there is any mistake or suggestion for improvement, please don't hesitate, come forward and take part in this Noble cause for future generations

Thanks,

Mirza Mubbashir Baig.

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Syllabus Overview

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$\sin(90^\circ - \theta) \equiv \cos\theta$, $\cos(90^\circ - \theta) \equiv \sin\theta$, $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta \equiv 1$.

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:	Notes and examples
• identify the forces acting in a given situation	e.g. by drawing a force diagram.
• understand the vector nature of force, and find and use components and resultants	Calculations are always required, not approximate solutions by scale drawing.
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero	Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact, and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.
• use Newton's third law.	e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4 Mechanics

4.2 Kinematics of motion in a straight line

Candidates should be able to:	Notes and examples
• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities	Restricted to motion in one dimension only. The term 'deceleration' may sometimes be used in the context of decreasing speed.
• sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that – the area under a velocity–time graph represents displacement, – the gradient of a displacement–time graph represents velocity, – the gradient of a velocity–time graph represents acceleration	
• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration	Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.
• use appropriate formulae for motion with constant acceleration in a straight line.	Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:	Notes and examples
• use the definition of linear momentum and show understanding of its vector nature	For motion in one dimension only.
• use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.	Including direct impact of two bodies where the bodies coalesce on impact. Knowledge of impulse and the coefficient of restitution is not required.

4 Mechanics

4.4 Newton's laws of motion

Candidates should be able to:	Notes and examples
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod	If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.
• use the relationship between mass and weight	$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value 10 (m s^{-2}) for g is expected.
• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration	Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.
• solve simple problems which may be modelled as the motion of connected particles.	e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:	Notes and examples
• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force	$W = Fd\cos\theta$; Use of the scalar product is not required.
• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae	
• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy	Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.
• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion	Including calculation of (average) power as $\frac{\text{Work done}}{\text{Time taken}}$. $P = Fv$.
• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.	

Topic 1

FORCES & EQUILIBRIUM

9709 Mathematics – (M1)

Topical Paper 4

Mirza Mubbashir Baig

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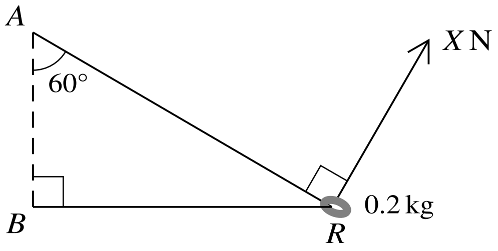
1 MJ 2023/P43/Q3



A smooth ring R of mass 0.2 kg is threaded on a light string ARB. The ends of the string are attached to fixed points A and B with A vertically above B. The string is taut and angle $ABR = 90°$. The angle between the part AR of the string and the vertical is 60°. The ring is held in equilibrium by a force of magnitude X N, acting on the ring in a direction perpendicular to AR (see diagram).

Calculate the tension in the string and the value of X. [5]

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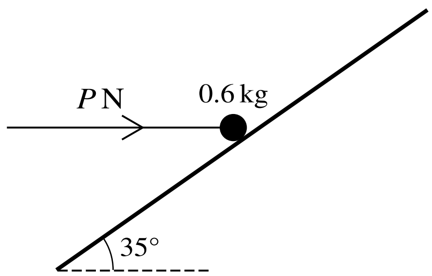
2 MJ 2023/P42/Q5



A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P. [6]

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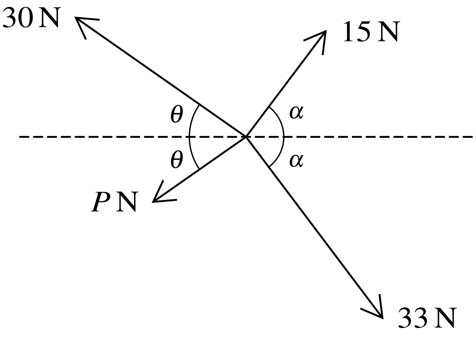
3 **MJ 2023/P42/Q3**



Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

(a) Show that $\left(\dfrac{14.4}{30-P}\right)^2 + \left(\dfrac{28.8}{P+30}\right)^2 = 1$. [4]

(b) Verify that $P = 6$ satisfies this equation and find the value of θ. [2]

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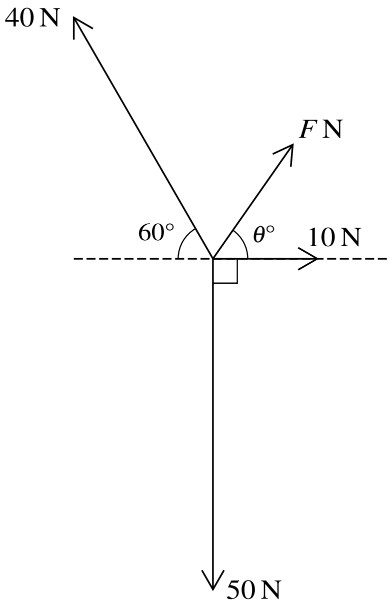
4 **MJ 2023/P41/Q5**



Four coplanar forces act at a point. The magnitudes of the forces are F N, 10 N, 50 N and 40 N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the value of F and the value of θ. [6]

(b) Given instead that $F = 10\sqrt{2}$ and $\theta = 45$, find the direction and the exact magnitude the resultant force. [3]

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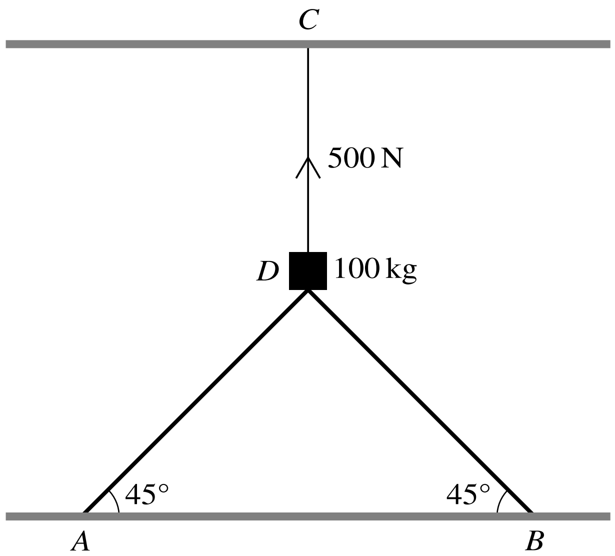
5 **FM 2023/P42/Q5**



The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD, each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

(a) Find the magnitude of the force in each of the struts AD and BD. [3]

A horizontal force of magnitude F N is applied to the block in a direction parallel to AB.

(b) Find the value of F for which the magnitude of the force in the strut AD is zero. [3]

---

6 **ON 2022/P43/Q7**

A particle P moves in a straight line through a point O. The velocity $v\,\mathrm{m\,s^{-1}}$ of P, at time t s after passing O, is given by

$$v = \frac{9}{4} + \frac{b}{(t+1)^2} - ct^2,$$

where b and c are positive constants. At $t = 5$, the velocity of P is zero and its acceleration is $-\frac{13}{12}\,\mathrm{m\,s^{-2}}$.

(a) Show that $b = 9$ and find the value of c. [5]

(b) Given that the velocity of P is zero only at $t = 5$, find the distance travelled in the first 10 seconds of motion. [5]

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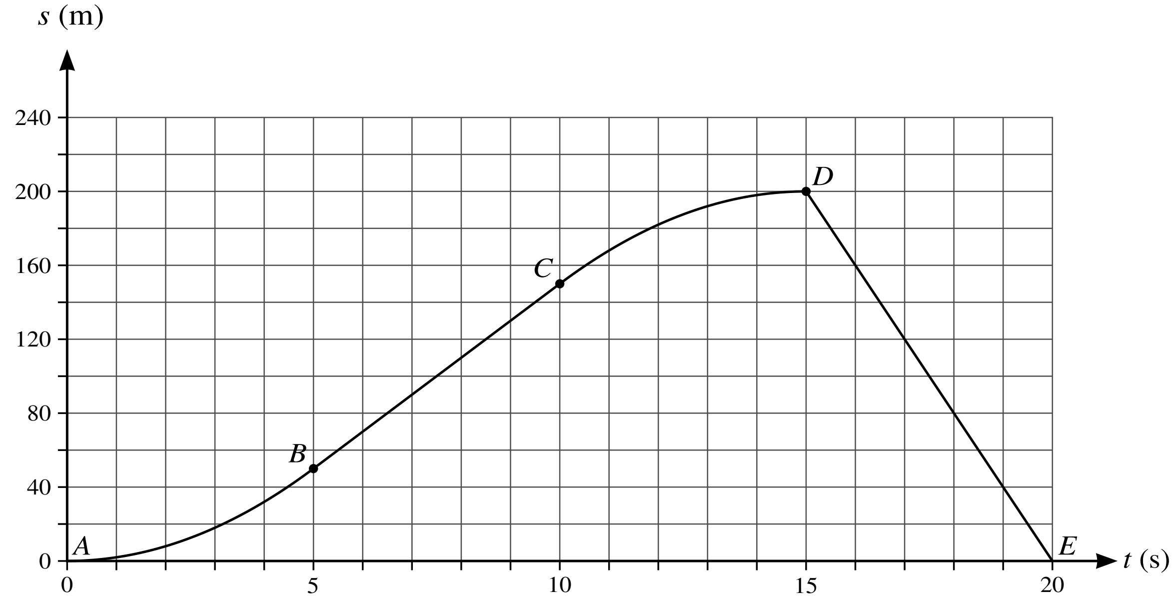
7 **ON 2022/P43/Q3**



The displacement of a particle moving in a straight line is s metres at time t seconds after leaving a fixed point O. The particle starts from rest and passes through points P, Q and R, at times $t = 5$, $t = 10$ and $t = 15$ respectively, and returns to O at time $t = 20$. The distances OP, OQ and OR are 50 m, 150 m and 200 m respectively.

The diagram shows a displacement-time graph which models the motion of the particle from $t = 0$ to $t = 20$. The graph consists of two curved segments AB and CD and two straight line segments BC and DE.

(a) Find the speed of the particle between $t = 5$ and $t = 10$. [1]

(b) Find the acceleration of the particle between $t = 0$ and $t = 5$, given that it is constant. [2]

(c) Find the average speed of the particle during its motion. [2]

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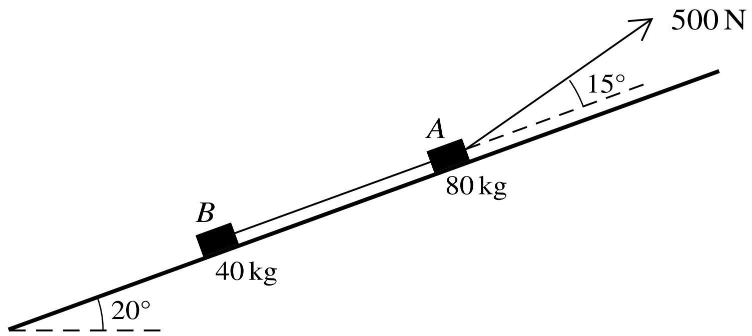
8 **ON 2022/P42/Q5**



A block A of mass 80 kg is connected by a light, inextensible rope to a block B of mass 40 kg. The rope joining the two blocks is taut and is parallel to a line of greatest slope of a plane which is inclined at an angle of 20° to the horizontal. A force of magnitude 500 N inclined at an angle of 15° above the same line of greatest slope acts on A (see diagram). The blocks move up the plane and there is a resistance force of 50 N on B, but no resistance force on A.

(a) Find the acceleration of the blocks and the tension in the rope. [5]

(b) Find the time that it takes for the blocks to reach a speed of $1.2\,\text{m}\,\text{s}^{-1}$ from rest. [2]

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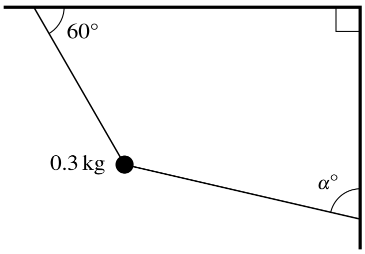
9 **ON 2022/P42/Q3**



A particle of mass 0.3 kg is held at rest by two light inextensible strings. One string is attached at an angle of 60° to a horizontal ceiling. The other string is attached at an angle $\alpha°$ to a vertical wall (see diagram). The tension in the string attached to the ceiling is 4 N.

Find the tension in the string which is attached to the wall and find the value of α. [6]

---

10 **ON 2022/P42/Q2**

A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$. [3]

A force of magnitude 7.2 N is now applied to P directly up a line of greatest slope of the plane.

(b) Given that P starts from rest, find the time that it takes for P to move 1 m up the plane. [4]

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11 **ON 2022/P41/Q6**

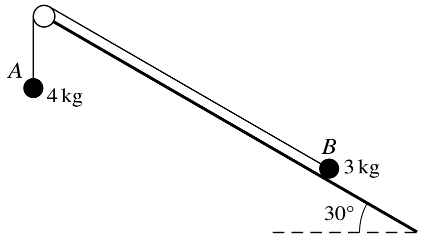

Fig. 6.1

Fig. 6.1 shows particles A and B, of masses 4 kg and 3 kg respectively, attached to the ends of a light inextensible string that passes over a small smooth pulley. The pulley is fixed at the top of a plane which is inclined at an angle of 30° to the horizontal. A hangs freely below the pulley and B is on the inclined plane. The string is taut and the section of the string between B and the pulley is parallel to a line of greatest slope of the plane.

(a) It is given that the plane is rough and the particles are in limiting equilibrium.

Find the coefficient of friction between B and the plane. [6]

(b)

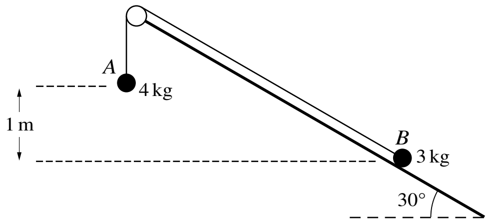

Fig. 6.2

It is given instead that the plane is smooth and the particles are released from rest when the difference in the vertical heights of the particles is 1 m (see Fig. 6.2).

Use an energy method to find the speed of the particles at the instant when the particles are at the same horizontal level. [6]

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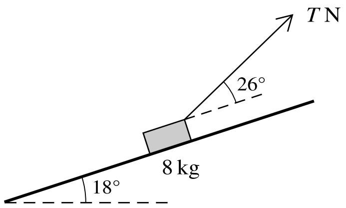
12 **ON 2022/P41/Q4**

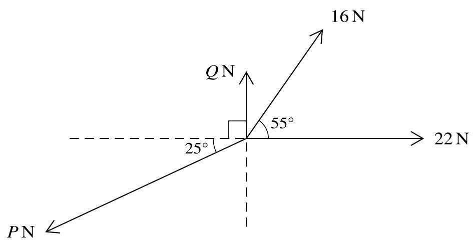


A block of mass 8 kg is placed on a rough plane which is inclined at an angle of 18° to the horizontal. The block is pulled up the plane by a light string that makes an angle of 26° above a line of greatest slope. The tension in the string is T N (see diagram). The coefficient of friction between the block and plane is 0.65.

(a) The acceleration of the block is $0.2\,\mathrm{m\,s^{-2}}$.
Find T. [7]

(b) The block is initially at rest.
Find the distance travelled by the block during the fourth second of motion. [2]

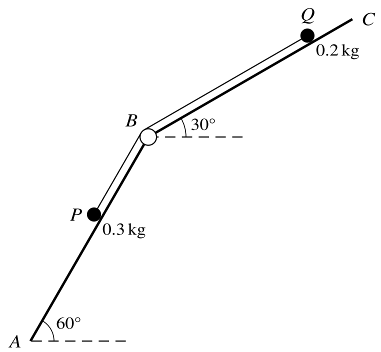
13 **ON 2022/P41/Q1**



Coplanar forces of magnitudes P N, Q N, 16 N and 22 N act at a point in the directions shown in the diagram. The forces are in equilibrium.

Find the values of P and Q. [5]

14 **MJ 2022/P43/Q6**

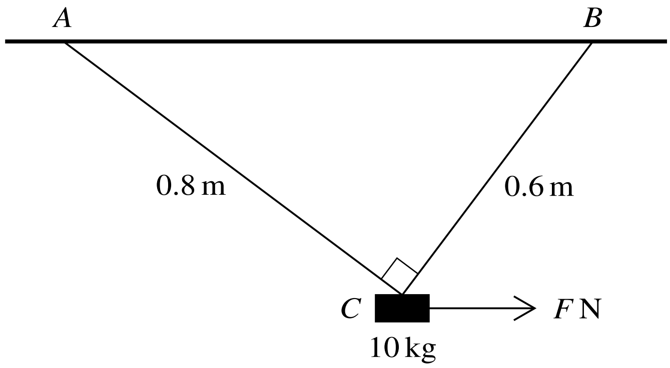


Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. P lies on a smooth plane AB which is inclined at 60° to the horizontal. Q lies on a plane BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that the plane BC is smooth and that the particles are released from rest.
Find the tension in the string and the magnitude of the acceleration of the particles. [5]

(b) It is given instead that the plane BC is rough. A force of magnitude 3 N is applied to Q directly up the plane along a line of greatest slope of the plane.
Find the least value of the coefficient of friction between Q and the plane BC for which the particles remain at rest. [5]

15 **MJ 2022/P43/Q4**



The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90°$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. [5]

(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. [3]

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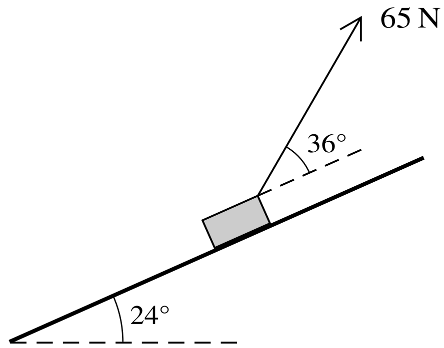
16 **MJ 2022/P42/Q5**



A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is μ. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find μ. [6]

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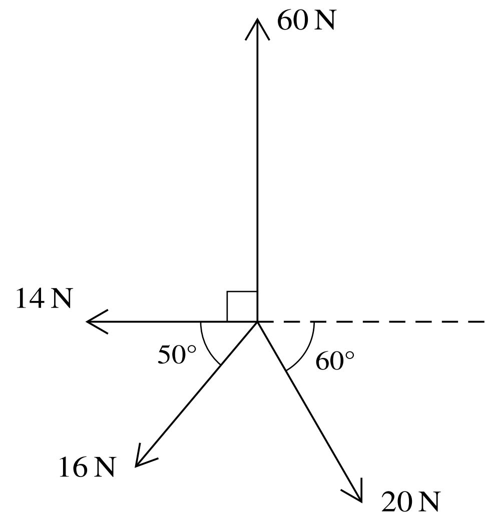
17 **MJ 2022/P42/Q2**



Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the resultant force. [6]

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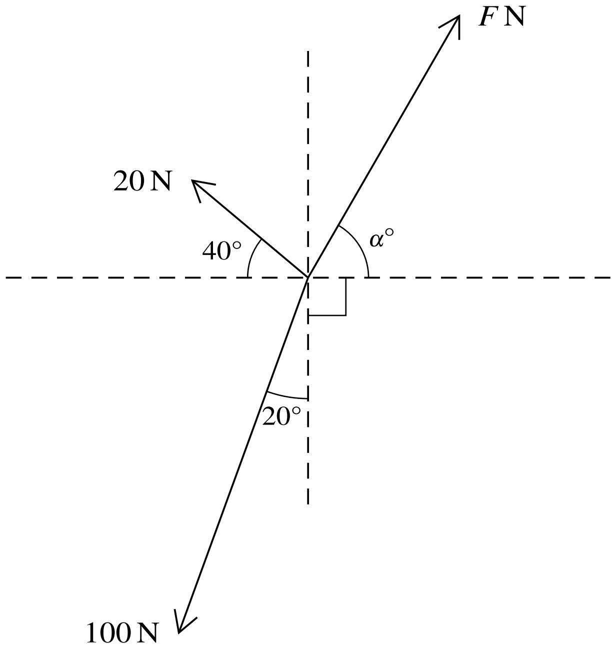
18 **MJ 2022/P41/Q4**



Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α. [6]

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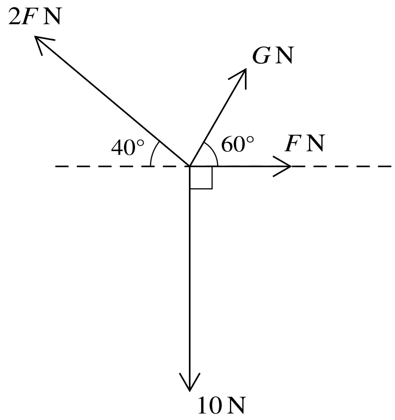
19 **MJ 2022/P41/Q3**

A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of X for which the crate remains at rest. [5]

20 **FM 2022/P42/Q5**



Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and $2F$ N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the values of F and G. [5]

(b) Given instead that $F = 3$, find the value of G for which the resultant of the forces is perpendicular to the 10 N force. [2]

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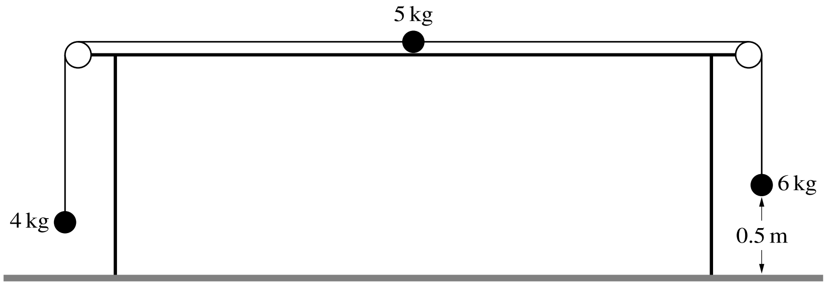
21 **ON 2021/P43/Q6**



The diagram shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.

(a) Show that the coefficient of friction between the 5 kg particle and the table is 0.4. [2]

The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest.

(b) Find the acceleration of the 4 kg particle and the tensions in the strings. [5]

(c) In the subsequent motion the 8 kg particle hits the ground and does not rebound.

Find the time that elapses after the 8 kg particle hits the ground before the other two particles come to instantaneous rest. (You may assume this occurs before either particle reaches a pulley.) [5]

---

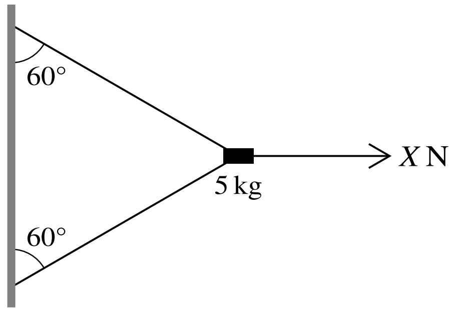
22 **ON 2021/P43/Q2**

A particle of mass 8 kg is suspended in equilibrium by two light inextensible strings which make angles of 60° and 45° above the horizontal.

(a) Draw a diagram showing the forces acting on the particle. [1]

(b) Find the tensions in the strings. [6]

23 **ON 2021/P42/Q6**



A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

(a) Given that $X = 100$, find the tension in the lower string. [4]

(b) Find the least value of X for which the block remains in equilibrium in the position shown. [4]

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24 **ON 2021/P41/Q4**

A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to prevent the particle sliding down the plane. The coefficient of friction between the particle and the plane is 0.35.

(a) Draw a sketch showing the forces acting on the particle. [1]

(b) Find the least possible value of P. [5]

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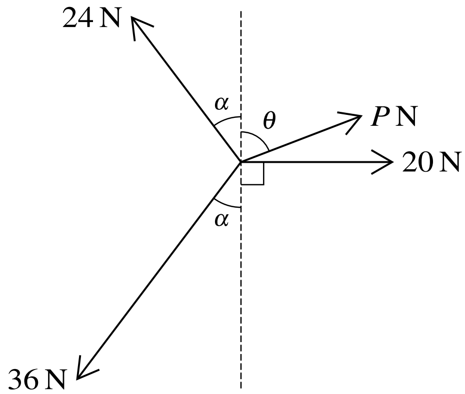
25 **ON 2021/P41/Q3**



Coplanar forces of magnitudes 24 N, P N, 20 N and 36 N act at a point in the directions shown in the diagram. The system is in equilibrium.

Given that $\sin\alpha = \frac{3}{5}$, find the values of P and θ. [6]

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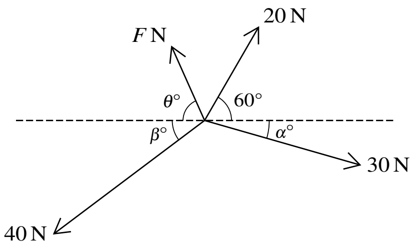
26 **MJ 2021/43/Q3**



Four coplanar forces act at a point. The magnitudes of the forces are 20 N, 30 N, 40 N and F N. The directions of the forces are as shown in the diagram, where $\sin\alpha° = 0.28$ and $\sin\beta° = 0.6$.

Given that the forces are in equilibrium, find F and θ. [6]

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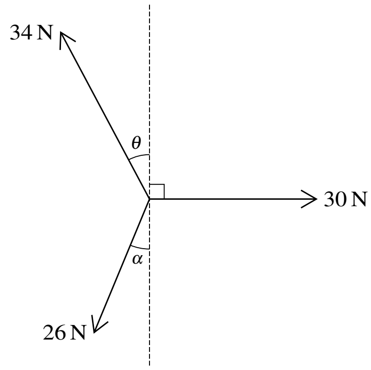
27 **MJ 2021/42/Q2**



Coplanar forces of magnitudes 34 N, 30 N and 26 N act at a point in the directions shown in the diagram.

Given that $\sin\alpha = \frac{5}{13}$ and $\sin\theta = \frac{8}{17}$, find the magnitude and direction of the resultant of the three forces. [6]

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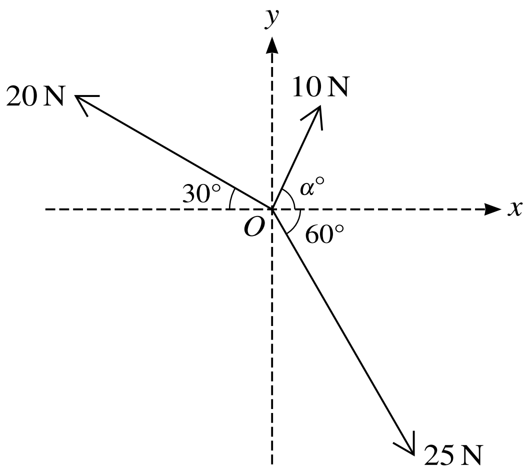
28 **9709/41/M/J/21/Q6**



Three coplanar forces of magnitudes 10 N, 25 N and 20 N act at a point O in the directions shown in the diagram.

(a) Given that the component of the resultant force in the x-direction is zero, find α, and hence find the magnitude of the resultant force. [4]

(b) Given instead that $\alpha = 45$, find the magnitude and direction of the resultant of the three forces. [5]

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29 **ON 2020/ P43/Q3**

A string is attached to a block of mass 4 kg which rests in limiting equilibrium on a rough horizontal table. The string makes an angle of 24° above the horizontal and the tension in the string is 30 N.

(a) Draw a diagram showing all the forces acting on the block. [1]

(b) Find the coefficient of friction between the block and the table. [5]

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30 **ON 2020/ P42/Q6**

A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ.

(a)

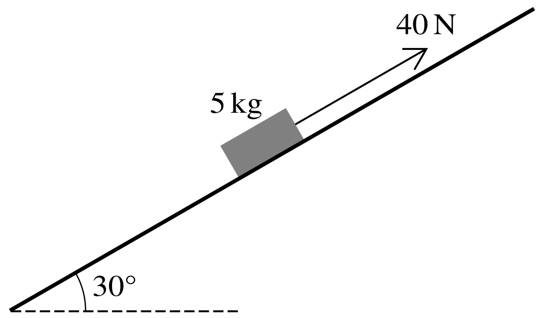

Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{\ }$

(b)

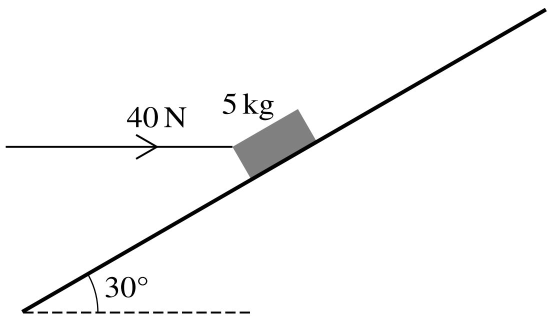

Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]
